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CALCULATION OF THE FOUR-QUARK CONDENSATES IN THE QCD MOTIVATED EXTENDED NAMBU-JONA-LASINIO MODEL [1]

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Abstract

The four-quark condensates appearing in QCD sum rule are calculated nonperturbatively in a realistic QCD motivated extended Nambu-Jona-Lasinio model. The calculation is in the framework of the effective potential for local composite operators up to next-to-the-leading order in the 1/N expansion in which the non-factorized parts of the condensates are included. We show in this paper the possibility of explaining the phenomenological enhancement factor $\kappa = 2.4$, needed for fitting the data in the factorization approximation, as the contribution of the non-factorized parts of the four- quark condensates.

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One of the interesting but difficult problems in nonperturbative QCD is the calculation of the quark and gluon condensates. The two-quark condensate breaks the chiral symmetry which explains the lightness of the low lying 0^- mesons, while all condensates are relevant to the well-known QCD sum rule [1]. QCD sum rule incorporates the quark and gluon condensates with perturbative QCD calculations. For example, in the study of the sum rule for mesons, the vacuum expectation value (VEV) of two current operators $< Tj^A(x)j^B(0) >$ is considered, and the operator product expansion (OPE) gives

$$i \int d^4x \, e^{iq\cdot x} \, < Tj^A(x)j^B(0) > = \sum_n C_n^{AB}(q) \, < O_n > \quad , \tag{1}$$

where $C_n^{AB}(q)$'s are perturbatively calculable Wilson coefficients and $< O_n >$'s are various nonperturbative vacuum condensates. In practical applications, on the RHS of (1), only $C_n^{AB}(q)$'s are calculated from perturbative QCD while the condensates $< O_n >$'s are left as free parameters determined by experimental inputs. To reduce the number of free parameters, people often make the simple $factorization$ approximation to express the four-quark condensate in terms of the two-quark condensate, i.e.

$$< \bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi > \approx \; F_\Gamma < \bar{\psi}\psi >^2 \quad , \tag{2}$$

where the Lorentz structure of Γ is 1 for scalar (S), $i\gamma_5$ for pseudoscalar (P), γ_μ for vector (V), $\gamma_5\gamma_\mu$ for axial-vector (A), $\sigma_{\mu\nu}$ for tensor (T) (for color and flavor non-singlet, the color and flavor group generators $\lambda_\alpha/2$, t_i should also be included), and F_Γ is a constant depending on the structure of Γ and can be easily calculated [1]. However, applications of QCD sum rule to various hadronic processes with the approximation (2), e.g. the sum rules for baryons [2] ρ-meson [3] and pseudoscalar meson [4], show that the theoretical results are not quite good and phenomenologically the $F_\Gamma < \bar{\psi}\psi >^2$ term should be enlarged by a factor $\kappa \approx 3.6$ for fitting the data [2][3][4]. Further study of the perturbative QCD corrections to the Wilson coefficient of the operator $(\bar{\psi}\Gamma\psi)^2$ shows that the $O(\alpha_s)$ correction increases the Wilson coefficient by a factor of 1.5 [5] which means that the enhancement factor should actually be $\kappa \approx 2.4$.

So far there is no theoretical explanation of the enhancement factor $\kappa \approx 2.4$. A possible and natural conjecture is that the need for an enhancement factor might be due to the neglect of the non-factorized part of the four-quark condensate $< \bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi >_{NF}$ in (2), but this is difficult to prove from the first principles of QCD. Several attempts have been made to discuss this problem. A careful calculation of the anomalous dimensions of the four-quark operators shows that the deviation from (2)

varies significantly with the renormalization scale, i.e. even if (2) is good at low energy it may fail at some high energy scale [6]. In Ref.[7], a general formalism of the effective potential for local composite operators in the framework of $1/N$ expansion was developed and was applied to calculate the four-fermion condensate $<\bar{\psi}\psi\bar{\psi}\psi>$ nonperturbatively in the Gross-Neveu model. It is shown that (2) *holds only in the* $N\to\infty$ *limit*, i.e. $<\bar{\psi}\psi\bar{\psi}\psi>_{NF}$ *is nonvanishing to order*-$1/N$. Up to order-$1/N$, for $N=3$, the numerical result shows that the ratio $R\equiv<\bar{\psi}\psi\bar{\psi}\psi>_{NF}/[F_S<\bar{\psi}\psi>^2]$ runs rapidly with the renormalization scale, and the minimal value of R is about $1/3$. This means that (2) is not a good approximation even at low energy. However, the Gross-Neveu model is a toy model which can not predict the value of κ to compare with the experiment. In recent years, there have been several papers studying QCD motivated Nambu-Jona-Lasinio (NJL) models as effective theories for low lying hadrons[8]. These models lead to quite successful phenomenological predictions, especially the extended Nambu-Jona-Lasinio (ENJL) model in Ref. [8], so that they may reflect some main features of nonperturbative QCD. Since the NJL model is a four-fermion interaction theory similar to the Gross-Neveu model, the method developed in Ref.[7] can be directly applied to it. In this paper, we take the EJNL model of Ref.[8] and calculate the four-quark condensates appearing in the QCD sum rule nonperturbatively by using the method of Ref.[7]. The four-quark condensates are generally related to the sum rule in the form $\sum\limits_{\Gamma}C_6^{\Gamma}<\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi>$ (cf. (1)). Once the condensates $<\bar{\psi}\psi>$ and $<\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi>$ are calculated, the enhancement factor κ can be obtained from

$$\kappa=\frac{\sum\limits_{\Gamma}C_6^{\Gamma}<\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi>}{\sum\limits_{\Gamma}C_6^{\Gamma}F_{\Gamma}<\bar{\psi}\psi>^2}=1+\frac{\sum\limits_{\Gamma}C_6^{\Gamma}<\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi>_{NF}}{\sum\limits_{\Gamma}C_6^{\Gamma}F_{\Gamma}<\bar{\psi}\psi>^2}\quad. \tag{3}$$

Here we sketch the main points of the method developed in Ref.[7]. To calculate the two-quark and four-quark condensates, we consider the following generating functional

$$\begin{aligned}Z[K_2,K_4]\equiv expiW[K_2,K_4]=i\int\mathcal{D}\psi\mathcal{D}\bar{\psi}\,expi\int d^4x\,\{\,\mathcal{L}+\sum_{\Gamma}K_2^{\Gamma}\bar{\psi}\Gamma\psi+\sum_{\Gamma}K_4^{\Gamma}\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi\\+P(K_2,K_4)\}\;,\end{aligned}\tag{4}$$

where K_2^{Γ} and K_4^{Γ} are local external sources, and $P(K_2,K_4)$ is a pure source term needed for satisfying the consistency condition requiring that no condensate effect occurs at the classical level [9]. Let W_c be the classical value of W . The consistency condition is [9]

$$\left.\frac{\delta^{n+m}W_c}{\delta(K_2^{\Gamma})^n\delta(K_4^{\Gamma})^m}\right|_{K_2,K_4=0}=0\quad. \tag{5}$$

Here, for simplicity, we have ignored the external sources for the single fields ψ and $\bar{\psi}$ in (4) since we are not calculating $<\psi>$ and $<\bar{\psi}>$ which actually vanish. Define the classical fields Σ_Γ and Ξ_Γ ,

$$\frac{\delta W}{\delta K_2^\Gamma} = \Sigma_\Gamma \ , \qquad \frac{\delta W}{\delta K_4^\Gamma} = \Sigma_\Gamma^2 + \Xi_\Gamma \ , \tag{6}$$

Σ_S and $\Xi_\Gamma + (1 - F_\Gamma)\Sigma_S^2$ give $<\bar{\psi}\psi>$ and $<\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi>_{NF}$, respectively, in the vacuum state. Once $W[K_2, K_4]$ is obtained, we can make the Legendre transformation

$$\Gamma[\Sigma, \Xi] = W[K_2, K_4] - \int d^4x \{ \sum_\Gamma K_2^\Gamma \Sigma_\Gamma + \sum_\Gamma K_4^\Gamma (\Sigma_\Gamma^2 + \Xi_\Gamma) \} \ , \tag{7}$$

to get the effective action $\Gamma[\Sigma, \Xi]$ and so the effective potential $V_{eff}(\Sigma, \Xi)$. The rules for calculating $W[K_2, K_4]$ are as follows. Define the well-known propagator [10]

$$i\mathcal{D}^{-1}(x, y; K_2) = \left. \frac{\delta^2 S}{\delta\psi(y)\delta\bar{\psi}(x)} \right|_{\psi,\bar{\psi}=0} \ , \tag{8}$$

where S is the action $S = \int d^4x \, [\mathcal{L} + \sum_\Gamma K_2^\Gamma \bar{\psi}\Gamma\psi + \sum_\Gamma K_4^\Gamma \bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi + P(K_2, K_4)]$. $\mathcal{D}$ is related to the physicsl propagator G by

$$G^{-1} = \mathcal{D}^{-1} - \Pi \ , \tag{9}$$

where Π is the proper self-energy of the ψ field. For models like the ENJL model, we can always consider a special proper self-energy diagram Π_s , composed of physical propagators and bare vertices, which is momentum-independent and is of the following structure [7]

$$\Pi_s = -i \sum_\Gamma b_\Gamma \Gamma \Delta_\Gamma \ , \tag{10}$$

where

$$\Delta_\Gamma \equiv - \frac{\delta W_L}{\delta K_2^\Gamma} = Tr(\Gamma G) \ , \tag{11}$$

and b_Γ is a constant determined by the model of interaction. In (11) W_L means the loop contribution to W . Define a new propagator G_s ,

$$G_s^{-1} = \mathcal{D}^{-1} - \Pi_s \ . \tag{12}$$

In terms of these notations, the formula for $W[K_2, K_4]$ derived in Ref. [7] is written as

$$\begin{aligned} W[K_2, K_4] = & \int d^4x \{ \mathcal{L} + \sum_\Gamma K_2^\Gamma \bar{\psi}\Gamma\psi + \sum_\Gamma K_4^\Gamma \bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi + P(K_2, K_4)\} - i \, Tr ln i G_s^{-1} \\ & - \int d^4x \sum_\Gamma b_\Gamma \Delta_\Gamma^2 - i < 0| exp i \int d^4x \, \mathcal{L}_I |0>'_{P2PI(\Pi_s)} \ , \end{aligned} \tag{13}$$

where $\mathcal{L}_I$ is the interaction Lagrangian, and the last term means the sum of all the partially two-particle irreducible diagrams with respect to Π_s (with propagators G_s 's and vertices in $\mathcal{L}_I$) defined in Ref. [7] except the type of diagrams shown in Fig.(4c) in Ref. [7]. The first term in (13) is the tree level contribution, while other terms are contributed by loop diagrams (W_L). In (13), Δ serves as an unknown constant which can be further solved from (11). Then the effective potential V_{eff} can be obtained from (13) and (7). The condensates can then be determined by the minimum of V_{eff} .

Actually, once W is obtained, we can also calculate the condensates simply from (6) by taking $K_2 = K_4 = 0$ without knowing V_{eff} . The solutions of (6) are the same as those obtained from the extremum condition of V_{eff} as it should be[7]. To pick up the true condensates (minimum of V_{eff}), one needs only calculate the values of V_{eff} for these solutions. This is much easier than calculating the complete function $V_{eff}(\Sigma, \Xi)$ for arbitrary Σ and Ξ .

The Lagrangian in the ENJL model in Ref. [8] is

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{\Lambda_\chi} + \mathcal{L}_{NJL}^{SP} + \mathcal{L}_{NJL}^{VA} , \tag{14}$$

where Λ_χ is a momentum cut-off, $\mathcal{L}_{QCD}^{\Lambda_\chi}$ is the QCD Lagrangian for momentum below Λ_χ , and

$$\begin{aligned}
\mathcal{L}_{NJL}^{SP} &= \frac{8\pi^2 G_S}{N_c \Lambda_\chi^2} \sum_{ab} (\bar{\psi}_R^a \psi_L^b)(\bar{\psi}_L^b \psi_R^a) \\
&= \frac{2\pi^2 G_S}{N_c N_f \Lambda_\chi^2} [(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \psi)^2] \\
&+ \frac{2\pi^2 \hat{G}_S}{N_c N_f \Lambda_\chi^2} \sum_{i=1}^{N_f^2-1} [(\bar{\psi} t_i \psi)(\bar{\psi} t_i \psi) + (\bar{\psi} i\gamma_5 t_i \psi)(\bar{\psi} i\gamma_5 t_i \psi)] ,
\end{aligned} \tag{15}$$

$$\begin{aligned}
\mathcal{L}_{NJL}^{VA} &= -\frac{8\pi^2 G_V}{N_c \Lambda_\chi^2} \sum_{ab} [(\bar{\psi}_L^a \gamma^\mu \psi_L^b)(\bar{\psi}_L^b \gamma_\mu \psi_L^a) + (\bar{\psi}_R^a \gamma^\mu \psi_R^b)(\bar{\psi}_R^b \gamma_\mu \psi_R^a)] \\
&= -\frac{4\pi^2 G_V}{N_c N_f \Lambda_\chi^2} [(\bar{\psi}\gamma^\mu \psi)(\bar{\psi}\gamma_\mu \psi) + (\bar{\psi}\gamma_5 \gamma^\mu \psi)(\bar{\psi}\gamma_5 \gamma_\mu \psi)] \\
&- \frac{4\pi^2 \hat{G}_V}{N_c N_f \Lambda_\chi^2} \sum_{i=1}^{N_f^2-1} [(\bar{\psi}\gamma^\mu t_i \psi)(\bar{\psi}\gamma_\mu t_i \psi) + (\bar{\psi}\gamma_5 \gamma^\mu t_i \psi)(\bar{\psi}\gamma_5 \gamma_\mu t_i \psi)] .
\end{aligned} \tag{16}$$

in which G_S , G_V are two coupling constants treated as free parameters, $\hat{G}_S \equiv N_f G_S$, $\hat{G}_V \equiv N_f G_V$, and the flavor group generator t_i is normalized as $t_i = \lambda_i/\sqrt{2}$, $i = 1, 2, \cdots, N_f^2 - 1$. It is argued in Ref. [8] that the NJL type Lagrangian $\mathcal{L}_{NJL}^{SP} + \mathcal{L}_{NJL}^{VA}$ may be understood as coming from integrating out the high momentum modes of quarks and gluons in the fundamental theory of QCD and is regarded as the main part in $\mathcal{L}_{QCD}$ that is responsible for the chiral symmetry breaking, while $\mathcal{L}_{QCD}^{\Lambda_\chi}$ is

responsible for the low-energy gluonic corrections to the broken chiral symmetry state. This supports the idea of the chiral quark model [11]. Even without the gluonic corrections from $\mathcal{L}_{QCD}^{\Lambda_\chi}$, the NJL type Lagrangian can give rather good phenomenological predictions for low energy hadron physics [8]. Since we are not aiming at accurate calculations, we simply neglect the gluonic corrections in this paper and keep only $\mathcal{L}_{NJL}^{SP} + \mathcal{L}_{NJL}^{VA}$ as the effective interactions between quarks, i.e. the Lagrangian $\mathcal{L}$ in (4) is taken to be

$$\mathcal{L} = \bar{\psi}\, i\partial\!\!\!/\, \psi + \mathcal{L}_{NJL}^{SP} + \mathcal{L}_{NJL}^{VA}\,. \tag{17}$$

The coefficients b_Γ in (10) and (13) are now

$$\begin{aligned} b_S^1 &= b_P^1 = \frac{4\pi^2 G_S}{N_c N_f \Lambda_\chi^2}\,, \qquad & b_S^{t_i} &= b_P^{t_i} = \frac{4\pi^2 \hat{G}_S}{N_c N_f \Lambda_\chi^2}\,, \\ b_V^1 &= b_A^1 = -\frac{8\pi^2 G_V}{N_c N_f \Lambda_\chi^2}\,, \qquad & b_V^{t_i} &= b_A^{t_i} = -\frac{8\pi^2 \hat{G}_V}{N_c N_f \Lambda_\chi^2}\,, \end{aligned} \tag{18}$$

and the pure source term $P(K_2, K_4)$ obtained from (5) is

$$P(K_2, K_4) = \frac{1}{2}\sum_\Gamma \left(\frac{(K_2^\Gamma)^2}{b_\Gamma^1 + b_\Gamma^{t_i} + 2K_4^\Gamma} \right). \tag{19}$$

There are three independent free parameters G_S, G_V, and Λ_χ in the ENJL model in Ref. [8]. They are related to the constituent chiral quark mass M_Q, the quark axial-vector coupling constant g_A, and the vector meson mass M_V by the following relations[8]

$$1/G_S = (M_Q/\Lambda_\chi)^2 \Gamma(-1, (M_Q/\Lambda_\chi)^2)(1 + \gamma_{-1})\,, \tag{20}$$

$$g_A = \frac{1}{1 + 4G_V (M_Q/\Lambda_\chi)^2 \Gamma(0, (M_Q/\Lambda_\chi)^2)(1 + \gamma_{01})}\,, \tag{21}$$

$$\Lambda_\chi^2 = \frac{2}{3} M_V^2 G_V \Gamma(0, (M_Q/\Lambda_\chi)^2)(1 + \gamma_{03})\,, \tag{22}$$

where $\Gamma(n-2, (M_Q/\Lambda_\chi)^2)$ is the incomplete gamma function [8], and γ_{-1}, γ_{01}, and γ_{03} are gluonic corrections which are actually not large and have been neglected in our Lagrangian (17). Apart from the gluonic corrections, the rest part of the formulae used to fit the data in Ref. [8] are all of the leading order in $1/N_c$ expansion. Actually M_Q, Λ_χ, and g_A are taken as the input parameters to fit the data in Ref. [8]. There are five different ways of fitting the data presented in Ref. [8], which determine different sets of values of M_Q, Λ_χ, and g_A, and the predictions are all successful. The

simplest one of the fits is their **Fit 4** in which $g_A = 1$. From (21) we see that this corresponds to $G_V = 0$ which will simplify our calculation. Moreover, it is explained by Weinberg [12] that g_A should actually be unity in the constituent quark model. In view of this, we take in this paper the set **Fit 4** in Ref. [8], and by means of (20)-(22), it leads to

$$G_S = 1.19, \qquad G_V = 0, \qquad \Lambda_\chi = 667\, MeV\,. \tag{23}$$

In the present model, $N_c = N_f = 3$. Let $N \equiv N_c = N_f$. Our dynamical calculation of the condensates is in the $1/N$ expasion. In doing so, we should take a proper limit of $N \to \infty$ such that the three terms in (17) are of the same order in this limit, i.e. both the propagation of the quark field and all the NJL type interactions are included. This means that (cf. (15)-(17)) we should treat the flavor singlet coupling constants G_S , G_V and the flavor non-singlet coupling constants $\hat{G}_S$, $\hat{G}_V$ as independent finite parameters as $N \to \infty$. After obtaining the condensates to certain order in $1/N$ expansion, we then take numerically $N = 3$ and $\hat{G}_S = 3G_S$, $\hat{G}_V = 3G_V$ when calculating the enhancement factor κ . In order to make comparison with the formulae in Ref. [8], we define the constituent chiral quark mass as

$$M_Q \equiv -b_S^1 < \bar{\psi}\psi >, \tag{24}$$

which corresponds to the mean-field approximation interpretation of M_Q. Using the method described above, similar to the calculation presented in Ref. [7], we obtain the following results.

In the large-N limit, $W[K_2, K_4]$ is contributed by chain diagrams with open ends. Then the gap equation (eq.(6) with $K_2 = K_4 = 0$) to this order reads

$$\begin{aligned} &< \bar{\psi}\psi >= -i\, tr \int_{\Lambda_\chi} \frac{d^4p}{(2\pi)^4} \frac{1}{\not{p} - M_Q} = -\frac{N^2 M_Q^3}{4\pi^2}\Gamma(-1, (M_Q/\Lambda_\chi)^2)\,, \\ &< \bar{\psi} i\gamma_5\psi >=< \bar{\psi}\gamma_\mu\psi >=< \bar{\psi}\gamma_5\gamma_\mu\psi >= 0\,. \end{aligned} \tag{25}$$

In (25) the $\int_{\Lambda_\chi}$ means that the Λ_χ -regulator used in Ref. [8] is taken for the momentum integration. In the chiral limit $< \bar{\psi}\psi >= N < \bar{u}u >$, thus we have

$$< \bar{u}u >= -\frac{N}{4\pi^2} M_Q^3\,\Gamma(-1, (M_Q/\Lambda_\chi)^2)\,. \tag{26}$$

which coincides with the result in Ref. [8]. It is also easy to see that, in the large-N limit, $[\delta W/\delta K_4^\Gamma]_{K_2,K_4=0} = \Sigma_\Gamma^2$, so that $[\Xi_\Gamma]_{K_2,K_4=0}$ vanishes. Since in this limit $F_\Gamma = 1$, $[\Xi_\Gamma]_{K_2,K_4=0}$ is just the non- factorized part of the four-quark condensate. Thus we have

$$< \bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi >_{NF} = 0 \tag{27}$$

in the large-N limit. The intuitive reason for the factorization of the four-quark condensate in the $N \to \infty$ limit is as follows. Diagrammatically, the four-quark condensate is obtained by removing a node from the chain diagram for W, which releases four quark lines. For open-end chains, the removal of a node causes two disconnected chains, each with two quark lines, and this is just the factorization of the four-quark condensate.

To order- $1/N$, we should further take into account the P2PI(Π_s) diagrams shown in Fig. 6 in Ref. [7], which include the closed chain diagrams. Removing a node from a closed chain does not cause two disconnected chains, so that factorization breaks down to this order. Up to Next-to-the- leading order in $1/N$ expansion, we see from (3) that only an order- $1/N$ calculation of $<\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi>_{NF}$ is further needed.

In practical applications, the currents in (1) are usually color octets. Similar to Ref.[7], by applying the method described above, we obtain the following up to order-$1/N$ results of the non-factorized parts of various relevant four-quark condensates.

$$\begin{aligned}
&< \bar{u}\lambda_\alpha u\bar{u}\lambda_\alpha u >_{NF} = (F_S/N)(S_{NF} - P_{NF} - A_{NF}) + O(1/N^2)\,,\\
&< \bar{u}\lambda_\alpha d\bar{d}\lambda_\alpha u >_{NF} = (F_S/N)S_{NF} + O(1/N^2)\,,\\
&< \bar{u}\lambda_\alpha u\bar{d}\lambda_\alpha d >_{NF} = 0 + O(1/N^2)\,,
\end{aligned} \tag{28}$$

$$\begin{aligned}
&< \bar{u}i\gamma_5\lambda_\alpha u\bar{u}i\gamma_5\lambda_\alpha u >_{NF} = (F_P/N)(S_{NF} - P_{NF} + A_{NF}) + O(1/N^2)\,,\\
&< \bar{u}i\gamma_5\lambda_\alpha d\bar{d}i\gamma_5\lambda_\alpha u >_{NF} = (F_P/N)S_{NF} + O(1/N^2)\,,\\
&< \bar{u}i\gamma_5\lambda_\alpha u\bar{d}i\gamma_5\lambda_\alpha d >_{NF} = 0 + O(1/N^2)\,,
\end{aligned} \tag{29}$$

$$\begin{aligned}
&< \bar{u}\gamma^\mu\lambda_\alpha u\bar{u}\gamma_\mu\lambda_\alpha u >_{NF} = (F_V/N)(S_{NF} + P_{NF} - \frac{1}{2}A_{NF}) + O(1/N^2)\,,\\
&< \bar{u}\gamma^\mu\lambda_\alpha d\bar{d}\gamma_\mu\lambda_\alpha u >_{NF} = (F_V/N)S_{NF} + O(1/N^2)\,,\\
&< \bar{u}\gamma^\mu\lambda_\alpha u\bar{d}\gamma_\mu\lambda_\alpha d >_{NF} = 0 + O(1/N^2)\,,
\end{aligned} \tag{30}$$

$$\begin{aligned}
&< \bar{u}\gamma_5\gamma^\mu\lambda_\alpha u\bar{u}\gamma_5\gamma_\mu\lambda_\alpha u >_{NF} = (F_A/N)(S_{NF} + P_{NF} + \frac{1}{2}A_{NF}) + O(1/N^2)\,,\\
&< \bar{u}\gamma_5\gamma^\mu\lambda_\alpha d\bar{d}\gamma_5\gamma_\mu\lambda_\alpha u >_{NF} = \frac{F_A}{N}S_{NF} + O(1/N^2)\,,\\
&< \bar{u}\gamma_5\gamma^\mu\lambda_\alpha u\bar{d}\gamma_5\gamma_\mu\lambda_\alpha d >_{NF} = 0 + O(1/N^2)\,,
\end{aligned} \tag{31}$$

$$
\begin{aligned}
&< \bar{u}\sigma^{\mu\nu}\lambda_\alpha u\bar{u}\sigma_{\mu\nu}\lambda_\alpha u >_{NF} = (F_T/N)(S_{NF} - P_{NF}) + O(1/N^2)\,,\\
&< \bar{u}\sigma^{\mu\nu}\lambda_\alpha d\bar{d}\sigma_{\mu\nu}\lambda_\alpha u >_{NF} = (F_T/N)S_{NF} + O(1/N^2)\,,\\
&< \bar{u}\sigma^{\mu\nu}\lambda_\alpha u\bar{d}\sigma_{\mu\nu}\lambda_\alpha d >_{NF} = 0 + O(1/N^2)\,,
\end{aligned}
\tag{32}
$$

where

$$
\begin{aligned}
F_S &= -F_P = -\frac{1}{2} + O(1/N^2)\,,\\
F_V &= -F_A = -2 + O(1/N^2)\,,\\
F_T &= -6 + O(1/N^2)\,,
\end{aligned}
\tag{33}
$$

and

$$
\begin{aligned}
S_{NF} &= -\frac{i}{4}\frac{\pi^2 G_S}{N\Lambda_\chi^2}\int_{\Lambda_\chi}\frac{d^4q}{(2\pi)^4}\frac{T_S^2(q^2,M_Q)}{1-\frac{\pi^2 G_S}{N\Lambda_\chi^2}T_S(q^2,M_Q)}\,,\\
P_{NF} &= -\frac{i}{4}\frac{\pi^2 G_S}{N\Lambda_\chi^2}\int_{\Lambda_\chi}\frac{d^4q}{(2\pi)^4}\frac{T_P^2(q^2,M_Q)}{1-\frac{\pi^2 G_S}{N\Lambda_\chi^2}T_P(q^2,M_Q)}\,,\\
A_{NF} &= -\frac{i}{4}\frac{\pi^2 G_S}{N\Lambda_\chi^2}\int_{\Lambda_\chi}\frac{d^4q}{(2\pi)^4}\frac{q^{-2}T_{PA}(q^2,M_Q)T_{AP}(q^2,M_Q)}{1-\frac{\pi^2 G_S}{N\Lambda_\chi^2}T_P(q^2,M_Q)}\,,
\end{aligned}
\tag{34}
$$

in which

$$
\begin{aligned}
T_S(q^2,M_Q) &= 4iN\int_{\Lambda_\chi}\frac{d^4p}{(2\pi)^4}\,tr\,[\frac{1}{\not p+\not q/2-M_Q}\frac{1}{\not p-\not q/2-M_Q}]\\
&= \frac{NM_Q^2}{\pi^2}\,\Gamma(-1,\,(\frac{M_Q}{\Lambda_\chi})^2) - (2q^2-8M_Q^2)\,I(q^2,\,M_Q)\,,\\
T_P(q^2,M_Q) &= 4iN\int_{\Lambda_\chi}\frac{d^4p}{(2\pi)^4}\,tr\,[i\gamma_5\frac{1}{\not p+\not q/2-M_Q}i\gamma_5\frac{1}{\not p-\not q/2-M_Q}]\\
&= \frac{NM_Q^2}{\pi^2}\,\Gamma(-1,\,(\frac{M_Q}{\Lambda_\chi})^2) - 2q^2\,I(q^2,\,M_Q)\,,\\
T_{PA}(q^2,M_Q) &= -T_{AP}(q^2,M_Q) = 4iN\int_{\Lambda_\chi}\frac{d^4p}{(2\pi)^4}\,tr\,[i\gamma_5\frac{1}{\not p+\not q/2-M_Q}\gamma_5\not q\frac{1}{\not p-\not q/2-M_Q}]\\
&= -i\,M_Q\,q^2\,I(q^2,\,M_Q)\,,\\
I(q^2,M_Q) &\equiv -\frac{N}{4\pi^2}\int_0^1 dx\,\Gamma(0,\,\frac{q^2(x^2-x)+M_Q^2}{\Lambda_\chi^2})\,.
\end{aligned}
\tag{35}
$$

In (35) tr is the trace in the Dirac spinor space. With all these, we can calculate the enhancement factor κ for practical processes.

Take the ρ-meson sum rule as an example. The relevant four-quark condensates are [3]

$$
\begin{aligned}
C_6 < O_6 >= 6\pi^3\alpha_s\{&< (\bar{u}\gamma^\mu\gamma_5\lambda_\alpha u - \bar{d}\gamma^\mu\gamma_5\lambda_\alpha d)^2 >\\
&+\frac{2}{9} < (\bar{u}\gamma^\mu\lambda_\alpha u + \bar{d}\gamma^\mu\lambda_\alpha d)\sum_{u,d,s}\bar{\psi}\gamma_\mu\lambda_\alpha\psi >\}.
\end{aligned}
\tag{36}
$$

Substituting (36) into (3), and using the results in (30) and (31), we obtain the enhancement factor for the ρ-meson sum rule

$$\kappa = 1 + \frac{1}{3}\frac{S_{NF}+P_{NF}}{<\bar{u}u>^2} + \frac{11}{42}\frac{A_{NF}}{<\bar{u}u>^2} + O(\frac{1}{N^2})\,. \tag{37}$$

When doing numerical calculations, we should bear in mind that the numbers in (23) are obtained from fitting the data with the large-N_c formulae together with gluonic corrections, while our present calculation is up to order-$1/N$ without gluonic corrections. Thus we should not take the numbers in (23) so seriously when calculating the value of κ . From (36) we see that the values of $S_{NF},\, P_{NF}$ and A_{NF} are only related to the two parameters G_S and M_Q^2/Λ_χ^2 which are further related to each other by (20). Therefore our κ is a function of only one parameter G_S . In **Table 1**, we list the values of $S_{NF},\, P_{NF}$ and A_{NF} for several values of G_S around the number given in (23). The corresponding numbers of κ in the ρ-meson sum rule are listed in **Table 2**. We see that in the small G_S region κ is a sensitive function of G_S , and $\kappa = 2.4$ is included in this region. At present G_S is not determined accuratly, so that we still cannot predict quantitatively if κ is precisely 2.4. However, due to this sensitivity, the possibility of having $\kappa = 2.4$ mainly depends on the positivity of the order-$1/N$ contribution in (37) (the sum of the second and third terms in (37)) which is not obvious in general. Our calculation shows that it is really the case.

Inview of the fact that κ is sensitive to the value of G_S which is not yet well determined, our conclusion in this paper is that *the explanation of* $\kappa = 2.4$ *as the contributions of the non-factorized parts of the four-quark condensates is possible.*

References

1. M.A. Shifman, A.I. Vainstein, and V.I. Zakharov, Nucl.Phys.**B147**(1979) 385,448,519

2. See for example H.G. Dosch, Univ.Heidelberg preprint HD-THEP-85-13; Y. Chung, H.G. Dosch, M. Kremer, and D. Schall, Z.Phys.**C25**(1984)151

3. G. Launer, S. Narison, and R. Tarrach, Z.Phys.**C26**(1984)476

4. M. Kremer, N.A. Papadopoulos, and K. Schilcher, Phys,Lett.**143B**(1984)476

5. L.V. Lanin, V.P. Spiridonov, and K.G. Chetykin, Yad. Fiz.**44**(1986)1372

6. M. Jamin and M. Kremer, Nucl.Phys.**B227**(1986)349

7. H.J. He and Y.P. Kuang, Z.Phys.**C47**(1990)565

8. J. Bijnens, C. Bruno, and E. de Rafael, Nucl.Phys.**B390**(1993)501 and references therein.

9. Y.P. Kuang, X. Li, and Z.Y. Zhao, Commun. Theor. Phys.**3**(1984)251; H.J. He, Q. Wang, and Yu-Ping Kuang, Z.Phys.**C45**(1990)427

10. R. Jackiw, Phys.Rev.D**9**(1974)1686

11. A. Manohar and H. Georgi, Nucl.Phys.**B234**(1984)232

12. S. Weinberg, Phys.Rev..Lett.,**65**(1990)1181

Table 1 Values of S_{NF} , P_{NF} , and A_{NF} (in the unit of $< \bar{u}u >^2$) for given values of G_S .

G_s	2.0	1.5	1.3	1.25	1.2	1.15	1.1
S_{NF}	0.00069	0.0028	0.0079	0.011	0.016	0.026	0.047
P_{NF}	0.010	0.017	0.026	0.030	0.037	0.048	0.072
A_{NF}	-0.00054	-0.00077	-0.00094	-0.00099	-0.0011	-0.0011	-0.0012
G_s	1.08	1.05	1.05	1.02	1.01	1.009	1.008
S_{NF}	0.065	0.12	0.23	0.37	0.85	0.96	1.1
P_{NF}	0.090	0.15	0.26	0.40	0.88	0.99	1.1
A_{NF}	-0.0012	-0.0013	-0.0013	-0.0014	-0.0014	-0.0014	-0.0014
G_s	1.007	1.006	1.005	1.004	1.003	1.002	1.001
S_{NF}	1.3	1.5	1.9	2.4	3.4	5.3	12
P_{NF}	1.3	1.6	1.9	2.5	3.4	5.3	12
A_{NF}	-0.0014	-0.0014	-0.0014	-0.0014	-0.0014	-0.0014	-0.0014

Table 2 Values of the enhancement factor κ (eq.(37)) for given values of G_S .

G_s	2.0	1.5	1.3	1.25	1.2	1.15	1.1
κ	1.0035	1.0065	1.011	1.014	1.017	1.024	1.039
G_s	1.08	1.05	1.03	1.02	1.01	1.009	1.008
κ	1.051	1.089	1.16	1.26	1.57	1.65	1.74
G_s	1.007	1.006	1.005	1.004	1.003	1.002	1.001
κ	1.86	2.0	2.3	2.6	3.3	4.6	8.7